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MATHEMATICAL MODEL OF SEISMOACOUSTIC MONITORING OF BLAST FIELDS FOR REMOTE RECONNAISSANCE

This article discusses the construction of a mathematical model of seismoacoustic monitoring of explosion fields for remote exploration. As is known, seismoacoustic monitoring of blast fields is used for remote sensing and is a set of routine observations, whereby the mode of the observations themselves and the spectral parameters of the object under study depend on the research task at hand. As shown in the article, the reliability of explosive signal classification is improved through the use of remote sensing information technologies based on seismoacoustic monitoring. To achieve the research goal, it is necessary to construct a mathematical model of a continuous explosive field signal that would reflect the most important aspects of the explosive field signal monitoring process. In the process of constructing such a model, it is necessary to take into account both the parameters describing the process itself and the parameters of interference and natural background noise, as well as the characteristics of the transmission function of the medium.

To monitor explosive fields, it is necessary to collect statistical data for various explosive field signals and transfer functions of the media in which the signal propagates. This will provide a priori information about both the explosive field at the study points and the explosive field signals themselves, which will significantly reduce its impact on the evaluation of the studied explosive field signal. The work takes into account the influence of the instability of the parameters of the studied process and optimizes the procedure for processing the observed data according to criteria that take into account the characteristics of natural background interference. It is shown that the process of monitoring explosive fields boils down to the evaluation of informative parameters of parametric mathematical models of individual and continuous signals of the explosive field, the superposition of which forms the explosive field itself. The set of all informative parameters of each signal of the explosive field forms a vector of these parameters in n -dimensional Euclidean space. The optimal estimation of signal parameters involves determining the vector of free parameters that minimizes the value of the consistency criterion between the model and the observation data. Such a model provides good consistency in the case of modeling a linear system of oscillatory objects and, thus, takes into account the oscillatory nature of explosive signals. Thus, the article presents a new mathematical model of the explosive seismic field, which takes into account different types of signals in the explosive field, and provides a mathematical apparatus for solving this model. To assess the adequacy of the model for non-separable signals in the explosive field, simulation modeling of non-separable signals was carried out within the framework of an improved seismoacoustic monitoring methodology based on seismoacoustic analysis.

Keywords: seismoacoustic monitoring; parametric mathematical model; seismoacoustic analysis; explosive fields; seismoacoustic signal; seismoacoustic signal model.

Introduction

Seismoacoustic monitoring of blast fields for remote sensing is a set of regime observations, and the regime of the observations themselves and the spectral parameters of the object under study depend on the research task. Increasing the reliability of blast signal classification is achieved through the use of information technologies in remote sensing based on seismoacoustic monitoring. The term "regime observations" used above provides a clear procedure.

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First of all, it is necessary to build a mathematical model of the inseparable signal of the blast field, which would reflect the most significant moments of the process of monitoring blast field signals. In the process of building such a model, it is necessary to take into account both the parameters that describe the process itself and the parameters of the interference and natural noise background, as well as the specifics of the environment's transfer function. To monitor blast fields, it is necessary to collect statistics for various signals of blast fields and transfer functions of the environments in which the signal propagates. This will provide a priori ideas about both the explosive field at the study points and the explosive field signals themselves. A priori knowledge of the random process of interference will enable a significant reduction in its influence on estimating the explosive field signal under study. This weakening is achieved by optimizing the processing procedures that take into account the a priori statistics of the random interference process. The work takes into account the influence of the instability of the research process parameters and optimizes the procedure for processing observed data according to criteria that consider the characteristics of the natural background interference. The process of monitoring explosive fields is reduced to estimating the informative parameters of parametric mathematical models of separate and inseparable signals of the explosive field, the superposition of which forms the explosive field itself. The set of all informative parameters of each explosive field signal forms a vector of these parameters in n -dimensional Euclidean space. Each of these vectors uniquely characterizes the signal, the parameters of the mathematical model of which are this vector. Thus, the classification of resolved and inseparable signal explosion fields is reduced to the classification of these vectors in n -dimensional Euclidean space.

Review of recent research and publications. Today, issues related to seismic acoustic monitoring and the impact of shock waves on objects are described in the works of domestic and foreign researchers, namely: Mostovoy V.S., Vyzhva S.A., Shcherbina S.V., Kendzera O.V., Gabriel Albert (a French scientist who studied shock waves and worked on their mathematical description), Gaizim Walzman (Garry J. Walzmann - American physicist specializing in shock physics and explosive phenomena), John Solomon (John Solomon - scientist working in the field of shock wave physics and materials mechanics), Kevin Truscott (Kevin M. Truscott - engineer and scientist who studies the interaction of shock waves with structures), Leonardo Bassani (Leonardo Bassani - Italian scientist who studied the physics of explosions and shock waves), James R. Melville (James R. Melville – a physicist specializing in the study of shock phenomena and materials mechanics).

The purpose of this article is to construct a mathematical model of an indistinguishable explosive field signal that would reflect the most important aspects of the explosive field signal monitoring process. In the process of constructing such a model, it is necessary to take into account both the parameters describing the process itself and the parameters of interference and natural background noise, as well as the characteristics of the transmission function of the medium.

Main part

Mathematical model of indivisible signals. The model of the separate signal of the explosive field is a superposition of solutions to a second-order differential equation, which describes the superposition of oscillators that enter at different times, each with its own amplitude and natural frequency. The model of the explosive field we will present is the model of separate signals in the following way:

$$M(t, \lambda) = \sum_{i=0}^I \Phi(t - \lambda_{0+4i}) \lambda_{1+4i} [e^{-\lambda_{2+4i}(t - \lambda_{0+4i})} \sin[\lambda_{3+4i}(t - \lambda_{0+4i})]]; = \{\lambda_{p+4i}\}_{i=\overline{0, I}; p=\overline{0, 4}}, \quad (1)$$

here λ – vector of free model parameters;

I – number of pilmoles that are in superposition;

p – corresponding submodel number;

$\Phi(t)$ – Heaviside function;

i – submodel serial number;

λ_{0+4i} – signal arrival time;

l_{1+4i} – signal amplitude;

l_{2+4i} – signal attenuation coefficient;

l_{3+4i} – signal frequency.

The optimal estimation of signal parameters involves determining the vector of free parameters that minimizes the value of the criterion of agreement between the model and the observation data. Such a model gives good agreement in the case of modeling a linear system of oscillating objects and thus takes into account the oscillatory nature of explosive signals.

As a criterion of agreement, we choose the value of the norm L_2 (the integral of the square of the deviation of the model from the observed data $y(t)$) or L_1 (the integral of the modulus of the deviation of the model from the observed data $y(t)$). In the first case, the criterion $F(\lambda)$ takes the form $F(\lambda) = \int_T [y(t) - M(t, \lambda)]^2 dt$.

In another case: $F(\lambda) = \int_T |y(t) - M(t, \lambda)| dt$

The optimal estimate of free parameters λ^* is the point in the parameter space that minimizes $F(\lambda)$:

$$F(\lambda^*) = \min_{\lambda \in \Lambda} F(\lambda), \quad (2)$$

$y(t)$ – analytical approximation vector of a function of observed data values;

Λ – the set of all possible values of a vector λ .

1. These parameters enter the model nonlinearly. As a result, criterion (2) is a surface with many local extrema. This type of criterion has determined the approach to finding the global minimum. The points in the vicinity of which local extrema are located are selected by the random search method (Monte Carlo) according to a priori distributions [2]. Therefore, the number of calculation cycles must be sufficiently large to ensure the correctness of the result, which becomes increasingly complex with a large model dimensionality. Or it would be best to have good a priori ideas about the expected result. In conclusion, I would like to present the best parameters for the object under study.

To find the local minimum of the criterion, we need to calculate the time derivatives $\frac{\partial F(\lambda)}{\partial \lambda_k}$,

$k = \overline{0, K}$, and, equating them to zero, create a system of equations that has the form:

$$\frac{\partial F(\lambda)}{\partial \lambda_k} = \int_T [y(t) - M(t, \lambda)] \frac{\partial M(t, \lambda)}{\partial \lambda_k} dt = 0, \quad k = \overline{0, K}$$

Система приводиться:

$$\int_T [y(t) D(M(t, \lambda))] dt = \int_T [M(t, \lambda) D(M(t, \lambda))] dt,$$

here $D(M(t, \lambda))$ – a vector composed of functions, each of which is a derivative of the model with respect to all corresponding components of the vector λ .

For model (1), this vector has the form:

$$D(M(t, \lambda)) = \left\{ \frac{\partial M(t, \lambda)}{\partial \lambda_{p+i}} \right\}; \quad p = \overline{0, 3}; \quad i = \overline{0, I}. \quad (3)$$

The relevance of the variational approach in solving geophysical problems can be traced, for example, in [3].

The search for the minimum of criterion (2) is carried out using the Levenberg-Marquardt algorithm [4,5], for a priori randomly selected points in the space of free parameters of model (1).

Model (1) can be generalized to the case of a flow of inseparable signals, i.e., when the signal carriers intersect. Then the model of a flow of inseparable signals takes the form:

$$M(t, \lambda) = \sum_{s=1}^S \sum_{i=0}^I \Phi(t - \lambda_{0+i,s}) \lambda_{1+i,s} [e^{-\lambda_{2+i,s}(t - \lambda_{0+i,s})} \sin [\lambda_{3+i,s}(t - \lambda_{0+i,s})]] , \quad (4)$$

here S - the number of signals that overlap in the group;

λ - matrix of free model parameters;

I - number of pilmoles that are in superposition;

p - corresponding submodel number;

$\Phi(t)$ - Heaviside function;

i - submodel serial number.

$\lambda_{0+i,s}$ - start time of the S - th signal;

$\lambda_{1+i,s}$ - amplitude of the S - th signal;

$\lambda_{2+i,s}$ - attenuation coefficient of the S - th signal;

$\lambda_{3+i,s}$ - frequency of the S - th signal.

That is, the model (4) consists of S separate discrete signals. In the general case, S is a random variable. In the matrix of free parameters of the model $L = \{\lambda_{k,s}\}; k = \overline{0, K}; s = \overline{1, S}; K = 4I$; , the column with the number S is the vector of parameters of the S -th signal.

In this parameter vector $\text{mod}(k, 4)$ - is the number of the $\lambda_0, \lambda_1, \lambda_2, \lambda_3$ parameters. This harmonic has the number $\text{ant}(k, 4)$. We adopt the following notation: $\text{mod}(k, 4)$ - is the remainder of the number divided k by 4, and $\text{ant}(k, 4)$ - is the integer part of the division k by 4.

The above example of model (1) can be considered a special case of the more general model (4), in the case when $\lambda_{0+i,s}$ does not depend on i .

The optimal estimate of the matrix of free parameters of the model (4) Λ is determined by the values of the free parameters that give the minimum value of criterion (2) on the set of local minima of this criterion. The procedure for obtaining the optimal estimate consists in determining the points of local extrema closest to each of them for the set of M matrices Λ , and then selecting the global one on the set of local minima.

The point that gives this minimum criterion in the $(K+1) \cdot S$ dimensional space is chosen as optimal for the free parameters of the model signal. In such a procedure, convergence to the optimal solution is ensured by probability with increasing number M . In this case, we can assume that we have obtained as a result of the solution a superposition of separate signals M (1) that began at different times.

Assessment of the adequacy of the model of inseparable signals of the blast field. To assess the adequacy of model (4), we will conduct a simulation of the explosion field by implementing a stochastic process at each observation point. The simulation was conducted in the Mathcad 6.0 software environment.

To solve this problem, it is necessary to simulate both the microseismic background that accompanies the observation and the superposition of signals that are inseparable from each other, which simulate the explosion field signals. The explosion field signals and interference signals are simulated by Berlage signals, because the physically feasible signal is well approximated by Berlage pulses [6]:

$$y(t) = \eta(t) e^{-\alpha t} \sin(\omega t). \quad (5)$$

To take into account the leading edge of the explosive field signal, the Berlage pulse generalization can be represented in the following form:

$$y(t) = A\eta(t-\tau)(t-\tau)^\beta \exp\{-\alpha(t-\tau)\} \sin[\omega(t-\tau)]. \quad (6)$$

here $\eta(t-\tau)$ – is the Heaviside function, for the Berlage impulse, which entered at time τ ;

$y(t)$ – observed data;

$\{A, \tau, \alpha, \omega, \beta\}$ vector of free model parameters;

A - amplitude of oscillations;

τ - start time of the signal;

α - attenuation coefficient of the signal;

ω - frequency of the signal;

β - parameter that characterizes the signal front.

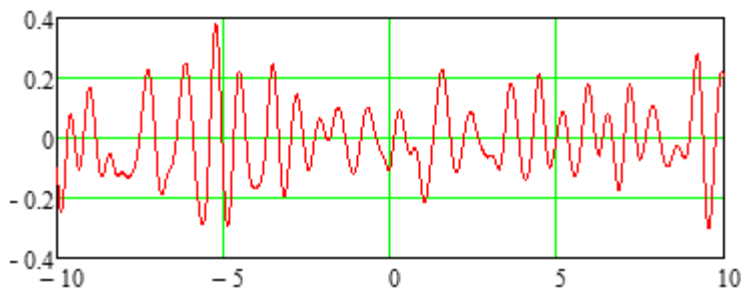


Fig. 1. Fragment of simulation modeling of the microseismic background. The abscissa axis represents time in seconds, while the ordinate axis indicates the amplitude of the vibration acceleration in relative units

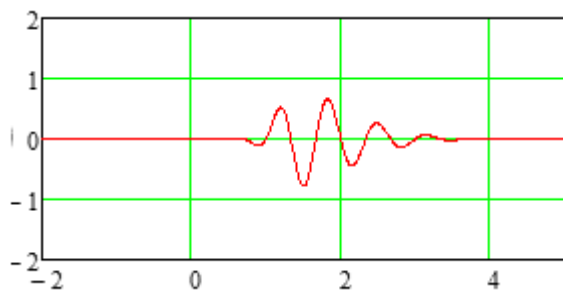


Fig. 2. Simulation of the 1st signal of the explosion field by a Berlage pulse with an amplitude of 0.8 relative units. The abscissa axis represents time in seconds, and the ordinate axis represents the amplitude of the acceleration of oscillations in relative units

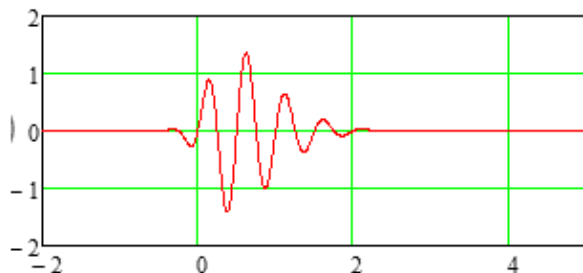


Fig. 3. Simulation of the 2nd signal of the explosive field by a Berlage pulse with an amplitude of 1.2 relative units. The abscissa axis represents time in seconds, and the ordinate axis represents the amplitude of the acceleration of oscillations in relative units

According to the conducted research, the microseismic background is simulated in the form of a superposition of 30 generalized Berlage pulses (6) with amplitudes not exceeding 0.4 relative units (fig. 1) [7, 8].

To assess the adequacy of model (4), we will construct three inseparable signals that simulate the signals of the blast field by generalized Berlage pulses (6) with amplitudes of 0.8, 1.2, and 1.5 in relative units (fig. 2-6).

Three inseparable signals were selected for visualization purposes.

Fig. 2 presents the result of the simulation of the 1st signal of the explosive field by a Berlage pulse with an amplitude of 0.8 relative units, a frequency of 0.6 Hz, and a decrement of 0.28. The abscissa axis represents time in seconds, and the ordinate axis represents the amplitude of the acceleration of oscillations in relative units.

In fig. 3, the result of simulation modeling of the 2nd signal of the explosive field by the Berlage pulse with an amplitude of 1.2 relative units, a frequency of 0.8 Hz, and a decrement of 0.36 is presented. The abscissa axis represents time in seconds, and the ordinate axis represents the amplitude of the acceleration of oscillations in relative units.

Fig. 4 presents the result of the simulated 3rd signal of the explosive field by the Berlage pulse with an amplitude of 1.5 relative units, a frequency of 0.7 Hz, and a decrement of 0.25. The abscissa axis represents time in seconds, and the ordinate axis represents the amplitude of the acceleration of oscillations in relative units.

Figure 5 presents the results of simulation modeling of three inseparable signals from the explosive field, simulated using Berlage pulses with amplitudes of 1.5 (black curve), 0.8 (red curve), and 1.2 (green curve) relative units. The abscissa axis represents time in seconds, and the ordinate axis represents the amplitude of the acceleration of oscillations in relative units.

For simulating inseparable explosive signals against a microseismic background, modeling is performed as a superposition of three inseparable signals from the explosive field (fig. 5) and the microseismic background (fig. 1). Fig. 6 presents the results of simulation modeling of three inseparable signals from the explosive field against a microseismic background using Berlage pulses. The amplitudes of the Berlage pulses that simulate the signals of the explosive field are 0.8, 1.2, and 1.5 relative units, and the amplitudes of the Berlage pulses that simulate the microseismic background do not exceed 0.4 relative units.

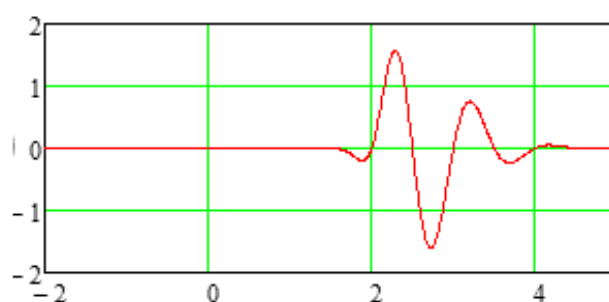


Fig. 4. The result of simulation modeling of the 3rd signal of the explosive field xy by the Berlage pulse with an amplitude of 1.5 relative units. The abscissa axis represents time in seconds, and the ordinate axis represents the amplitude of the acceleration of oscillations in relative units

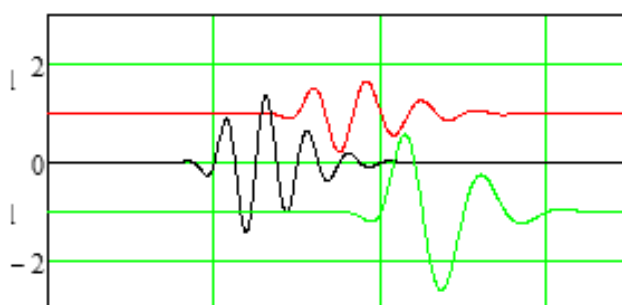


Fig. 5. Results of simulation modeling of three inseparable signals of the explosive field simulated by Berlage pulses with amplitudes of 1.5 (black curve), 0.8 (red curve), and 1.2 (green curve) relative units. The abscissa axis represents time in seconds, and the ordinate axis represents the amplitude of the acceleration of oscillations in relative units

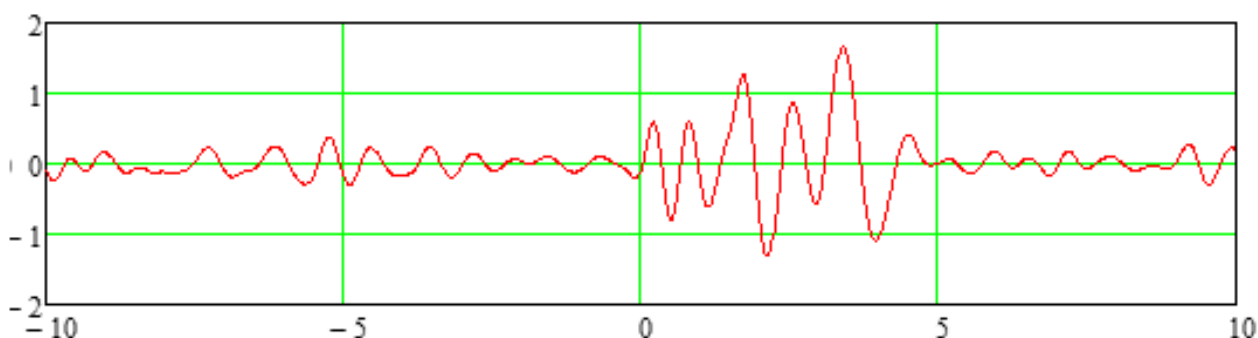


Fig. 6. Simulation modeling of the superposition of three inseparable signals of the explosion field against the background of a microseismic background. The abscissa axis represents time in seconds, and the ordinate axis represents the amplitude of the acceleration of oscillations in relative units

The result of solving model (6) for three simulated signals of the explosion field by Berlage pulses with amplitudes of 0.8, 1.2, and 1.5 relative units, against the background of a microseismic background simulated as a superposition of 30 Berlage pulses with amplitudes not exceeding 0.4 relative units, is presented in fig. 7. To obtain the parameters of the recorded explosion signals in the data shown in Figure 6, model (4) is employed. To solve model (4), which is a superposition of separate signals (1), model (1) is solved for each separate signal, while all other signals are considered as noise. In this example, model (1) was solved for each of the three se-parate signals of the explosive field, which are additively included in the inseparable signal, as shown in Figure 6. The result of the first signal is a signal with an amplitude of 0.79, a frequency of 0.63 Hz, and a decrement of 0.28. The result of the second signal is a signal with

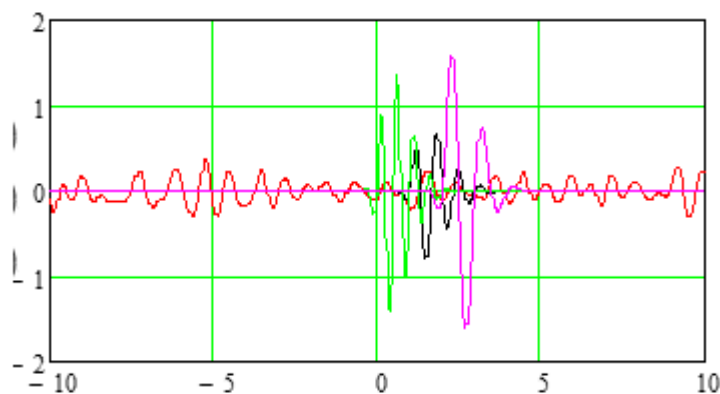


Fig. 7. Result of simulation of model (3.19) (signals of the explosion field with Berlage pulses with amplitudes of 1.22 (green curve), 0.79 (black curve), 1.57 (purple curve) relative units against the background of the microseismic background). The abscissa axis represents time in seconds, and the ordinate axis represents the amplitude of the acceleration of oscillations in relative units

an amplitude of 1.22, a frequency of 0.82 Hz, and a decrement of 0.33. The result of the third signal is a signal with an amplitude of 1.57, a frequency of 0.72 Hz, and a decrement of 0.23.

To solve the model (4), which is a superposition of separate signals, each separate signal of the explosive field (1) is considered separately, while all others are considered as noise.

Thus, the following conclusion can be made: the adequacy of the model (4) for the simulated non-separable signal does not exceed 5%.

Conclusions

A new mathematical model of the explosive seismic field is presented, which accounts for various types of signals in the explosive field. A new mathematical model of non-separable signals of the explosive field is presented. A mathematical apparatus for solving this model is provided. It is worth noting that these algorithms were developed for automated seismic acoustic monitoring systems [9].

To assess the adequacy of the model for non-separable signals in the explosive field, simulation modeling of non-separable signals was carried out within the framework of an improved methodology for seismoacoustic monitoring, based on seismoacoustic analysis.

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*І. О. Ярмолай***МАТЕМАТИЧНА МОДЕЛЬ СЕЙСМОАКУСТИЧНОГО
МОНІТОРИНГУ ПОЛІВ ВИБУХУ ДЛЯ ДИСТАНЦІЙНОЇ РОЗВІДКИ**

У даній статті розглядається побудова математичної моделі сейсмоакустичного моніторингу полів вибуху для дистанційної розвідки. Як відомо, сейсмоакустичний моніторинг вибухових полів використовується для дистанційного зондування і є сукупністю режимних спостережень, причому режим самих спостережень і спектральні параметри досліджуваного об'єкта залежать від поставленого дослідницького завдання. Підвищення надійності класифікації вибухових сигналів, як показано в статті, досягається за рахунок використання інформаційних технологій дистанційного зондування на основі сейсмоакустичного моніторингу. Для досягнення мети дослідження необхідно побудувати математичну модель нерозрізненого сигналу вибухового поля, яка б відображала найважливіші моменти процесу моніторингу сигналів вибухового поля. У процесі побудови такої моделі необхідно враховувати як параметри, що описують сам процес, так і параметри перешкод і природного фонового шуму, а також особливості передавальної функції середовища.

Для моніторингу вибухових полів необхідно зібрати статистичні дані для різних сигналів вибухових полів і передавальних функцій середовищ, в яких поширюється сигнал. Це дасть апіорні уявлення як про вибухове поле у точках дослідження, так і про самі сигнали вибухового поля, що дозволить значно зменшити його вплив на оцінку досліджуваного сигналу вибухового поля. У роботі враховується вплив нестабільності параметрів досліджуваного процесу та оптимізується процедура обробки спостережуваних даних за критеріями, що враховують характеристики природних фонових перешкод. Показано, що процес моніторингу вибухових полів зводиться до оцінки інформативних параметрів параметричних математичних моделей окремих і нерозривних сигналів вибухового поля, суперпозиція яких утворює саме вибухове поле. Сукупність усіх інформативних параметрів кожного сигналу вибухового поля утворює вектор цих параметрів у n -вимірному евклідовому просторі.

Оптимальна оцінка параметрів сигналу передбачає визначення вектора вільних параметрів, що мінімізує значення критерію узгодженості між моделлю та даними спостереження. Така модель забезпечує хорошу узгодженість у випадку моделювання лінійної системи коливальних об'єктів і, таким чином, враховує коливальний характер вибухових сигналів. Таким чином, у статті представлено нову математичну модель вибухового сейсмічного поля, яка враховує різні типи сигналів у вибуховому полі, наведено математичний апарат для розв'язання цієї моделі. Для оцінки адекватності моделі для нероздільних сигналів у вибуховому полі було проведено імітаційне моделювання нероздільних сигналів у рамках вдосконаленої методології сейсмоакустичного моніторингу, заснованої на сейсмоакустичному аналізі.

Ключові слова: сейсмоакустичний моніторинг; параметрична математична модель; сейсмоакустичний аналіз; вибухові поля; сейсмоакустичний сигнал; сейсмоакустична сигнальна модель.